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Instant Equilibrium Point and its Migration in Standing Tasks: Rambling and Trembling Components of the Stabilogram

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A method of decomposing stabilograms into two components, termed rambling and trembling, was developed. The rambling component reveals the motion of a moving reference point with respect to which the body's equilibrium is instantantly maintained. The trembling component reflects body oscillation around the reference point trajectory. The concepts of instant equilibrium point (IEP) and discrete IEP trajectory are introduced. The rambling trajectory was computed by interpolating the discrete IEP trajectory with cubic spline functions. The trembling trajectory is found as a difference between the approximated rambling trajectory and the COP trajectory. Instant values of the trembling trajectory are negatively correlated with the values of the horizontal ground reaction force at a zero time lag. It suggests that trembling is strongly influenced by a restoring force proportional to the magnitude of COP deviation from the rambling trajectory and acts without a time delay. An increment in relative COP position per unit of the restoring force, in mm/N, was on average 1.4 ± 0.4. The contribution of rambling and trembling components in the stabilogram was ascertained. The rambling variability is approximately three times larger than the trembling variability.

It is well established that the COP migration is a complex mix of the gravity line migration and inertial forces (Gurfinkel, 1973; King & Zatsiorsky, 1997; Murray et al., 1967; Thomas & Whitney, 1959; Winter, 1995). The gravity line (GL) is a vertical line passing through the total body's center of gravity (COG). Several methods of extracting the GL trajectory from posturographic recordings have been suggested (Benda et al., 1994; Caron et al., 1997; Eng & Winter, 1993; King & Zatsiorsky, 1997; Levin & Mizrahi, 1996; Shimba, 1984). Potentially these methods permit one to decompose a stabilogram into its static and dynamic components, with the first one depending on the instant position of the GL and the second one representing the acting inertial forces. The decomposition is useful for understanding posture biomechanics.

The goal of this study was to develop and explore a new method of stabilogram decomposition. In this paper we distinguish (a) the motion of a moving

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reference point (a set point or attracting point) with respect to which the body's equilibrium is instantantly maintained, a motion termed rambling; and (b) the oscillation of COP around the reference point trajectory, called trembling. We expect that this decomposition will be useful for exploring posture control mechanisms.

Basic Idea of the Method

The suggested method is based on the concept of the instant equilibrium point (IEP) or zero-force point. According to basic mechanics, equilibrium is the condition for which the resultant of all the forces acting on a given body is zero. If the body does not move in the vertical direction, and the sum of the external moments acting on the body is zero, the necessary and sufficient condition for equilibrium is $\Sigma F_{hor} = 0$, where F_{hor} are external forces acting on the body in the horizontal direction. When $\Sigma F_{hor} = 0$, the body is at equilibrium; it is either at rest or its center of mass moves with a constant velocity.

If the body deviates from the equilibrium position, restoring forces act to return the body to equilibrium. We call the position by which the equilibrium is maintained the *reference position*. The reference position is characterized by a reference point on a supporting surface (a set point or attracting point).

We define IEP as a COP position in an absolute system of coordinates at an instant when $\Sigma F_{hor} = 0$. At this moment, the COP coincides with the GL (see Zatsiorsky & King, 1998) and the body is instantly in a reference position. The consecutive positions of the IEP produce a *discrete IEP trajectory* (or discrete trajectory of the zero-force point). The discrete IEP trajectory represents discrete observations of the reference point trajectory.

The method is based on the following hypothesis: The continuous IEP trajectory obtained by a proper interpolation of the discrete IEP trajectory manifests the *rambling trajectory*, i.e., the continuous trajectory of the reference point. In this study, the discrete IEP trajectory was interpolated using cubic splines. Other methods of computing the continuous reference point trajectory from stabilographic recordings will be described in ensuing publications.

The hypothesis was tested in the following way: If the interpolated IEP trajectory mirrored a reference point trajectory, any deviation from the trajectory would have generated a restoring force that tended to return the object of control to the attracting trajectory. This hypothesis was tested in the experiment.

The Experiment

Ten healthy adults (8 men, 2 women) participated in this study. Average age was 28 ± 5 yrs, height was 179 ± 9 cm, and body mass was 78 ± 14 kg. No subjects had any known history of postural or skeletal disorder. Prior to testing, all subjects provided informed consent according to the Office of Regulatory Compliance of The Pennsylvania State University.

The subjects were asked to stand in an upright bipedal posture on a 40×90-cm force platform (model 4060S, Bertec Inc., Worthington, OH), considered hard surface. This testing was part of the experiment detailed in the accompanying paper in this issue (Duarte & Zatsiorsky, 1999). During testing, the subjects stood barefoot with eyes open. They were instructed to stand with feet a comfortable width apart (about shoulder width) and arms at sides. They were asked to maintain this same position throughout the trial. Only the data for the short periods of quiet standing are described in this paper (30 sec, open eyes, "stay quietly" instruction). The data are only presented for the anterior-posterior direction.

A code was written in MatLab software (MatLab 5.1, MathWorks Inc., Natick, MA) for data analysis. The program implements two sets of operations: decomposition of the stabilogram and analysis of the rambling and trembling trajectories. The first subprogram separates the rambling and trembling components of the COP trajectory by performing the following operations (Figure 1):

1. The instances of $F_{hor} = 0$ are identified in Figure 1a. The COP positions at these instances (IEP) are determined and interpolated by cubic spline functions to obtain an estimate of the rambling trajectory (Figure 1b). When using cubic splines, each segment between the data points is connected by a 3rd-order polynomial, and the slope of each cubic polynomial is matched at the data points (de Boor, 1978).
2. The COP trajectory is compared with the interpolated IEP trajectory, and the deviation of COP from the approximated rambling trajectory (relative COP position) is determined. This is the trembling trajectory (Figure 1c).

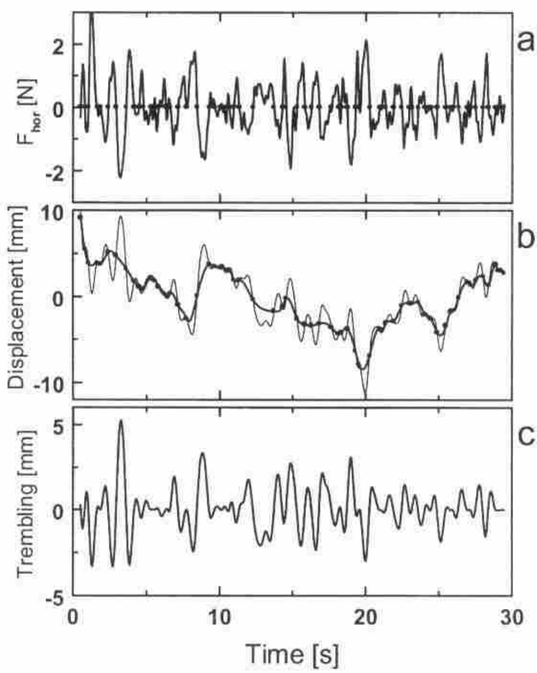

Figure 1 — Sequence of operations used for stabilogram decomposition. (a) F_{hor} data: instants of zero horizontal force (•). (b) In the COP displacement data (thin line), the COP positions at the instants $F_{hor} = 0$ (IEP) are located (•), and the spline interpolated rambling trajectory is shown (thick line). (c) Trembling trajectory, the difference between COP and interpolated rambling trajectories. Subject BD, male, age 25; Ht 1.69 m; Wt 74.8 kg.

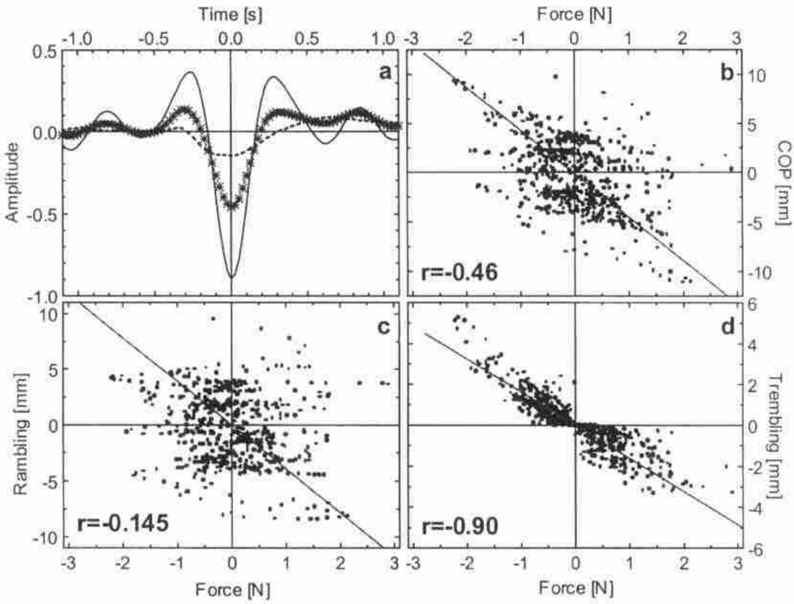

Figure 2 — Analysis of stabilogram decomposition. (a) Cross- correlation functions for F_{hor} vs. COP (-*-*-), F_{hor} vs. IEP (- - -), and F_{hor} vs. 'COP minus IEP' (—). (b) Instant COP positions (measured at fixed reference point) vs. instant values of F_{hor}, a scattergram. (c) Instant values of interpolated IEP position vs. those of F_{hor}, a scattergram. (d) Instant values of relative COP position (measured at IEP trajectory) vs. those of F_{hor}, a scattergram (same subject as in Figure 1).

The second subprogram performs the following operations:

1. The COP trajectory data as well as the trembling trajectory data are cross-correlated with the $F_{hor}(t)$, where $F_{hor}(t)$ is a time series of the horizontal force (Figure 2a). The cross-correlation was estimated using a maximal time lag of one second.
2. The absolute COP positions as well as the IEP trajectory (rambling trajectory) and the relative COP position (trembling trajectory) were cross-correlated with the F_{hor} data at a zero time lag (Figures 2b, c, and d, respectively). Because the X (F_{hor}) and Y (COP, rambling, and trembling) are random variables and cannot be classified as either dependent or independent, the reduced major axis model (RMA) was used for the linear regression (Sokal & Rohlf, 1981). In the RMA model, the slope (s_{RMA}) of the linear regression is computed as $s_{RMA} = s_{OLS} / |r_{XY}|$, where s_{OLS} is the slope for the ordinary least squares regression and r_{XY} is the correlation coefficient.

Routine statistical methods and the Fourier frequency analysis were employed to analyze the rambling and trembling time series data. Correlation coefficients were transformed to z values (Sokal & Rohlf, 1981) and a one-way ANOVA with a 0.05 significance level was conducted for testing hypothesis. The power spectral density (PSD) of the signal was estimated using Welch's averaged periodogram method (MathWorks, Inc., 1996).

Results

COP Position vs. F_{hor}

When instant values of absolute COP position (measured with respect to a fixed reference point) were cross-correlated with the instant values of the horizontal force, the negative correlation at a zero time lag was always observed (Figures 2a and 3a). The correlation coefficients were not large, on average –0.49 ± 0.13 (range = –0.63 to –0.25), but they were always larger than the correlation coefficients at the other time lags. King (1997) had observed similar negative correlations. The correlation between the rambling trajectory and F_{hor} was on average –0.25 ± 0.07 (range = –0.34 to –0.15). The correlation was significantly smaller than that between the F_{hor} and COP trajectories, $F(1, 18) = 25$, $p < 0.05$.

The correlation between trembling (relative COP position measured with respect to interpolated IEP trajectory) and F_{hor} was –0.85 ± 0.13, and ranged from –0.93 to –0.76. Because the experimental curves always pass through the origin of the system of coordinates (0,0 point), the two-dimensional distribution has a peculiar "butterfly" appearance and is evidently not Gaussian (e.g., Figure 2d). Thus the application of parametric tests of verifying statistical hypotheses is questionable in this case. When the non-normality of the distribution was neglected and routine methods were used to compare empirical correlation coefficients, the correlation was found to be significantly larger than the values for COP migration, $F(1, 18) = 225$, $p < 0.05$, and for the rambling, $F(1, 18) = 54$, $p < 0.05$ (Figure 3b). Note the extremely large values of F, 225 and 54, suggesting that the differences in magnitude of correlation are really noteworthy.

The regression equations for the trembling vs. F_{hor} data have a form $COP_{rel} = c \cdot F_{hor}$, where c is a coefficient having the dimensionality of m/N. Formally, c is similar to mechanical compliance. It characterizes the amount of deflection per unit of force. However, the coefficient cannot be called compliance because COP is not a physical object but rather just a coordinate of force application. The average value of c was equal to –1.4 ± 0.4 mm/N and ranged from –1.0 to –2.3 mm/N.

Quantification of Rambling and Trembling

The developed method allows a separate analysis of rambling, the migration of the reference point, and trembling, the COP oscillation around the moving reference point. Data on the variability of total COP migration, rambling, and trembling are presented in Table 1. Note that the sum of the percent variance of rambling and trembling is less than 100% of the COP variance (variance = SD^2, where SD = standard deviation). The reason for that is a slight positive correlation between rambling and trembling data (on average $r = 0.21 \pm 0.06$ at a zero time lag). Rambling *SD* is roughly three times larger than trembling *SD*.

The trembling frequency is largely determined by the time periods between zero-crossing points, i.e., the instances when horizontal force values and, correspondingly, the trembling trajectory intercept the zero line. On average, the zero-point-to-zero-point time was 0.41 ± 0.05 sec.

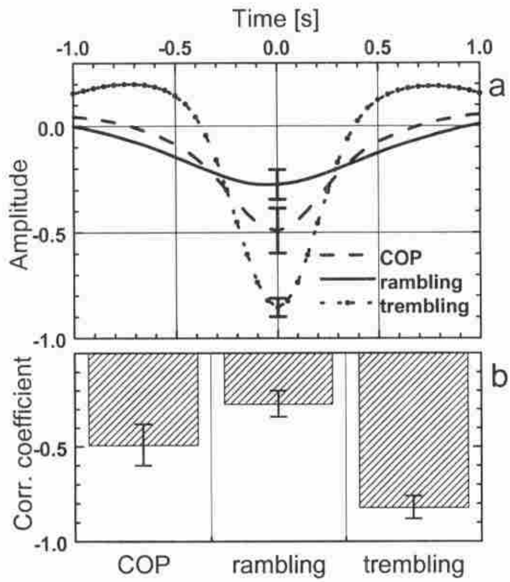

Figure 3 — (a) Cross-correlation between F_{hor} and COP, F_{hor} and rambling, F_{hor} and trembling, average curves for all subjects. Error bars (±1 *SD*) are for zero time lags. (b) Average correlation coefficient between F_{hor} and COP, F_{hor} and rambling, F_{hor} and trembling.

Table 1 Average COP, Rambling, and Trembling Data (*n* = 10)

	COP	Rambling	Trembling
SD (mm)	4.3 ± 1.9	3.9 ± 1.9	1.1 ± 0.4
SD (% COP)	100	90 ± 5	28 ± 9
Variance (% COP)	100	81 ± 9	9 ± 5
Maximal deviation (mm)	10.6 ± 3.9	8.7 ± 3.5	3.7 ± 1.3
Minimal deviation (mm)	–9.4 ± 3.2	–8.1 ± 3.1	–3.5 ± 1.1

The frequency bands of rambling and trembling are different (e.g., see Figure 4). The group average mean frequency (MF) of rambling was 0.16 ± 0.03 Hz, and the average MF of trembling was 0.67 ± 0.12 Hz, while the average MF for the COP time-series was 0.25 ± 0.07 Hz. Note the existence of the two peaks in the COP PSD, with the first one determined mainly by rambling and the second one by trembling (Figure 4). Hence the rambling/trembling decomposition works as a low/high-pass filter. However, the filter is specific: the rambling and trembling signals intercept at the instances of zero horizontal force.

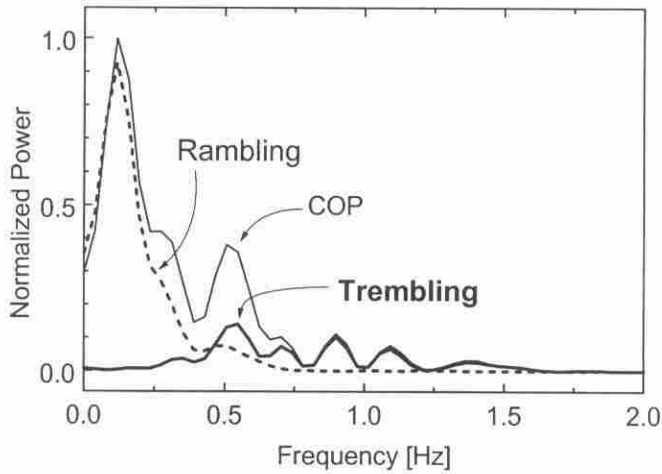

Figure 4 — Example of the PSD for COP, rambling, and trembling trajectories (same subject as in Figure 1). The first peak of the COP PSD is mainly determined by the rambling, the second peak by the trembling.

Discussion

The Restoring Forces in Trembling

The existence of a large negative correlation between relative COP position and F_{hor} means that a deviation of COP from the interpolated IEP trajectory is always associated with a horizontal force, tending to reduce the deviation. Hence the experimental evidence supports the main hypothesis of this study. The restoring force is proportional to the deviation and acts at a zero time lag. These features are typical for elastic forces that tend to return a system to a equilibrium. In physics, these systems are usually represented as parabolic (harmonic) potentials.

We can conclude that: (a) the interpolated IEP trajectory approximates a rambling trajectory, i.e., the trajectory of a moving point by which equilibrium is maintained; and (b) the restoring forces of trembling are largely similar to the elastic forces, and may indeed be the elastic forces.

Because trembling occurred due to the moment at the ankle joints, the values of the coefficient c can be transformed into ankle joint compliance or joint stiffness. Our finding seems to support the idea that small deviations from equilibrium during quiet standing are counterbalanced by the "apparent intrinsic stiffness" at the ankle joints (Gurfinkel et al., 1974). The restoration of equilibrium in this case does not require a reflex or voluntary intervention from the CNS. The suggested method might allow for quantification of the contribution of the quasi-elastic forces to maintain balance during trembling.

Determining the Rambling Trajectory

The discrete IEP trajectory represents discrete observations on the rambling trajectory, the trajectory of a reference point with respect to which the equilibrium of posture is maintained. The rambling trajectory between the discrete observations is unknown but should somehow be reconstructed. One attractive idea is to use the method of Latash (1993) in the framework of the λ-hypothesis. In this method, an instant equilibrium point at a joint is estimated as an intercept of the regression line of the instant values of the joint angle on the instant values of the joint torque. For the upright posture, a regression intercept of the COP position on the F_{hor} can be used as an estimate of the instant equilibrium point. Either a moving window technique or calculations from one IEP to another IEP (both at $F_{hor} = 0$) can be used. Whether these methods provide better estimates than the one in this study is questionable but they deserve to be explored.

In this study, we reconstructed the rambling trajectory simply by using cubic spline interpolation. The cubic splines by themselves have no physical meaning; they are just a mathematical technique. The only physical quality they posses is smoothness (if it is a physical quality). Using cubic splines, each segment between the data points is connected by a 3rd-order polynomial and the slope of each cubic polynomial is matched at the data points. Whether the real rambling trajectory is selected by the CNS to make the trajectory smooth is debatable. It seems plausible that the CNS uses a certain criterion of smoothness, e.g., a minimal jerk criterion (Flash & Hogan, 1985), and moves the reference point from one IEP to another in such a way that jerk is minimal.

Fortunately, due to the high frequency of the discrete IEP observations, the possible inaccuracies in estimating the continuous rambling trajectories from discrete IEP data seem not to be very large. At least they did not prevent high correlation between relative COP position and the restoring horizontal force.

Rambling, Trembling, COP, and GL Trajectories

The discovered facts are in agreement with the following scheme of the balance maintenance:

1. The CNS determines a reference point, or set point, by which the body's equilibrium is maintained. This concept is not new. The hypothesis about a reference used for equilibrium control was stated by Lestienne and Gurfinkel (1988) and confirmed by Gurfinkel et al. (1995) in dynamic posturography.
2. The reference point migrates and can be considered a moving attracting point. The hypothesis was inspired in part by the findings from our accompanying paper (Duarte & Zatsiorsky, 1999).
3. The body sways with respect to the reference point.
4. If the task is to maintain equilibrium, deflection from the rambling trajectory induces a restoring force. When the deflection is not too large, the restoring force is due to the "apparent intrinsic stiffness" of the muscles.

Similar to the rambling trajectory, GL trajectory during quiet stance coincides with COP trajectory when the horizontal force is zero. Hence IEP is the point where all four trajectories (rambling, trembling, COP, and GL) intercept. GL trajectory can be estimated by integrating the horizontal force twice on the interval from one IEP to the subsequent IEP (King & Zatsiorsky, 1997). Because the time intervals between consecutive IEPs are small, GL trajectory is located very close to rambling trajectory. However, the meanings of these trajectories and the methods of their calculation are different. Rambling trajectory is a latent motor control variable, a trajectory of the reference point for maintaining balance. It is estimated by interpolating the discrete IEP trajectory. GL trajectory is a physical (biomechanical) variable computed by integrating the F_{hor} twice over time. GL coincides with rambling trajectory only at IEP. During all other periods it deflects from the rambling trajectory. The interaction between the mentioned trajectories will be detailed in another paper.

Stabilogram Decomposition and λ-Hypothesis

Is the rambling trajectory identical to the equilibrium trajectory (virtual trajectory) proposed in the framework of the λ-hypothesis? The λ-hypothesis was introduced by Feldman (Asatryan & Feldman, 1965; Feldman, 1966a, 1966b) and further developed by several authors (see Latash, 1993, for details). To avoid terminological confusion and to appreciate Feldman's contribution in the development of the λ-hypothesis, we will call the equilibrium point as it is understood in the framework of the λ-hypothesis the *equilibrium point in the sense used by Feldman*, or *FEP* for short.

According to the λ-hypothesis, the FEP defines the joint compliance characteristics and, finally, an equilibrium position at the joint. Our definition of IEP is less restricted and is based on the balance of forces; the IEP trajectories measured in this paper may not correspond to the actual FEP trajectories. Hence our question is whether the IEP, or zero-force point (the position of the center of pressure on the support at the instant when the horizontal component of the ground reaction force equals zero), is just a specific manifestation of the FEP (a level of subthreshold membrane depolarization of a motoneuron pool to the agonistic muscles serving a joint).

A certain analogy seems apparent. In both cases, an equilibrium position is prescribed by the central command and the command actualization is achieved by peripheral mechanisms through a large contribution of the elastic or pseudo-elastic forces that are due to intrinsic muscle properties. The very idea of the rambling and trembling components of the stabilogram agrees well with this conception. The high linear relationships discovered between the deviation of the COP position from the rambling trajectory and the restoring force supports the idea that corrections in trembling are made "automatically" by the quasi-elastic forces that are due to intrinsic muscle properties. Still there are differences.

The first difference is conceptual. The IEP is an observed position of COP. It is a pure biomechanical concept that relies on straightforward mechanics, a momentary balance of forces. While interpretation of the IEP may vary, the very existence of IEP is indisputable. In standing, the GL and COP periodically crisscross at the instant equilibrium position. The FEP is a *virtual* point and its trajectory is defined as "a path that would be followed by a massless limb without damping and changes in external load" (Latash, 1993, p. 68). The concept of the FEP is similar to that of the rambling trajectory.

Contrary to the FEP, however, the IEP does not directly prescribe joint configurations. The human body can assume many different postures provided that the requirement for equilibrium, in the mechanical sense, is satisfied. Hence, once again contrary to the FEP, the IEP is not associated with any specific motoneuron pool. Still, if one considers only the general principle and not the details of the λ-hypothesis, our bias is that the findings of this study support the λ-hypothesis. In short:

1. The central command defines a reference point with respect to which the body equilibrium is maintained.
2. The reference point migrates following the rambling trajectory.
3. The body oscillates with respect to the rambling trajectory.
4. The restoring force in trembling is similar to an elastic force. The force is proportional to the deflection magnitude and occurs without a time delay. It is quite possible that this force is due to intrinsic muscle properties.
5. Periodically, the COP trajectory intersects the rambling trajectory. At this instant the body is at equilibrium momentarily, the horizontal projection of the ground reaction force is zero, and the IEP is registered.

This scheme corresponds well to the basic tenet of the λ-hypothesis: The central command determines an FEP, then the peripheral mechanisms that include the "apparent muscle stiffness" and the spinal reflexes take care of everything else.

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